SYDNEY GRAMMAR SCHOOL

NAME

MATHS MASTER

CANDIDATE NUMBER

2024 **Trial Examination**

Form VI Mathematics Advanced

Tuesday 13th August, 2024
8:40am

General Instructions

- Reading time — 10 minutes
- Working time — 3 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.
- A loose reference sheet is provided separate to this paper.
- Carefully remove the central staple.

Thirty Two Questions — 100 Marks

Section I (10 marks) Questions 1 – 10

- This section is multiple-choice. Each question is worth 1 mark.
- Record your answers on the provided answer sheet.

Section II (90 marks) Questions 11 – 32

- Relevant mathematical reasoning and calculations are required.
- Answer the questions in this paper in the spaces provided.

Collection

- Write your candidate number on this page and on the multiple choice sheet.
- Place everything inside this question booklet.

Checklist

- Reference sheet
- Multiple-choice answer sheet
- Candidature: 87 pupils

Writer: NWS

Section I

Questions in this section are multiple-choice.
Record the single best answer for each question on the provided answer sheet.

---

1. What is the domain of the function $y = \dfrac{1}{\sqrt{x}}$?

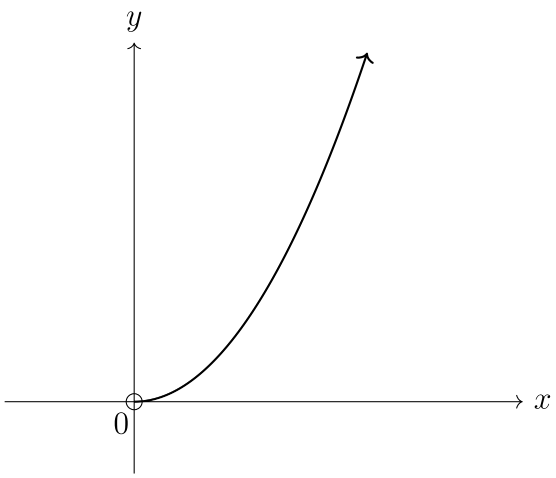
   (A) $(-\infty, 0)$

   (B) $(-\infty, 0]$

   (C) $[0, \infty)$

   (D) $(0, \infty)$

2. Which equation best represents the graph drawn below?



   (A) $y = 4x$, $x > 0$

   (B) $y = 4^x$, $x > 0$

   (C) $y = 4x^2$, $x > 0$

   (D) $y = \dfrac{4}{x}$, $x > 0$

3. A table of future value interest factors is shown below.

Table of future value interest factors

Period	*Interest rate per period*					
	1%	2%	3%	4%	6%	8%
2	2.0100	2.0200	2.0300	2.0400	2.0600	2.0800
3	3.0301	3.0604	3.0909	3.1216	3.1836	3.2464
4	4.0604	4.1216	4.1836	4.2465	4.3746	4.5061
5	5.1010	5.2040	5.3091	5.4163	5.6371	5.8666
6	6.1520	6.3081	6.4684	6.6330	6.9753	7.3359
7	7.2135	7.4343	7.6625	7.8983	8.3938	8.9228
8	8.2857	8.5830	8.8923	9.2142	9.8975	10.6366
9	9.3685	9.7546	10.1591	10.5828	11.4913	12.4876
10	10.4622	10.9497	11.4639	12.0061	13.1808	14.4866
11	11.5668	12.1687	12.8078	13.4864	14.9716	16.6455
12	12.6825	13.4121	14.1920	15.0258	16.8699	18.9771

An annuity involves making equal contributions of \$2500 into an account every quarter for 2 years at an interest rate of 4% per annum.
Based on the information provided, what is the future value of this annuity?

(A) \$5100

(B) \$20 714.25

(C) \$23 035.50

(D) \$26 591.50

4. What is the derivative of the function $f(x) = \log_e x^3$, with respect to x?

(A) $f'(x) = \dfrac{1}{3x}$

(B) $f'(x) = \dfrac{3}{x}$

(C) $f'(x) = \left(\dfrac{1}{x}\right)^3$

(D) $f'(x) = 3x$

5. Consider a dataset with mean 10 and standard deviation 2. Two scores, 10.5 and 9.5 are added to the dataset. Which choice shows how the mean and standard deviation are affected by the added scores?

   (A) Both mean and standard deviation will change.

   (B) Only standard deviation will decrease.

   (C) Only standard deviation will increase.

   (D) Both mean and standard deviation will stay the same.

6. Researchers are investigating population density, house size and the distance people live from the centre of a city. Data is plotted with distance on the horizontal axis.

   - For the graph of population density vs distance, the Pearson's correlation coefficient is $r = -0.563$.
   - For house size vs distance, the Pearson's correlation coefficient is $r = 0.357$.

   Given this information, which one of the following statements must be true?

   (A) Population density tends to be higher when it is farther away from the city centre.

   (B) House size tends to be larger when it is closer to the city centre.

   (C) House size is positively correlated with population density.

   (D) Population density is more strongly correlated with distance from the city centre than house size.

7. Suppose the graph of $y = f(x)$ has an x-intercept at 1, a y-intercept at 2, a minimum turning point $(5, -5)$ and a single horizontal asymptote $y = -4$.
   What can you say is definitely true about the graph of $y = -f(x-2)$?

   (A) Its x-intercept is -1.

   (B) Its y-intercept is -4.

   (C) It has a maximum turning point at $(7, 5)$.

   (D) Its horizontal asymptote cannot be determined.

8. A bag contains six white socks and eight black socks. Two socks are selected at random without replacement. Given the two selected socks are the same colour, which of the following is the closest to the probability that they are both black?

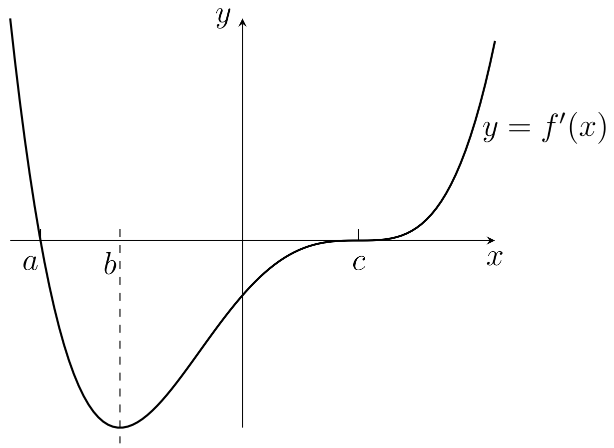
(A) 31%

(B) 54%

(C) 57%

(D) 65%

9. The graph of $y = f'(x)$ is drawn below.



Which of the following statements is true?

(A) $f(x)$ has a minimum turning point at $x = a$.

(B) $f(x)$ has a point of inflection at $x = b$.

(C) $f(x)$ has a maximum turning point at $x = c$.

(D) $f(x)$ has a point of inflection at $x = c$.

10. The function $f(x) = \sqrt{3}\cos(\alpha - 2x)$ is odd. Its graph also increases in the interval of $0 < x < \dfrac{\pi}{4}$. What is a possible value of α?

    (A) $-\dfrac{\pi}{2}$

    (B) 0

    (C) $\dfrac{\pi}{4}$

    (D) $\dfrac{\pi}{2}$

End of Section I

The paper continues in the next section

CANDIDATE NUMBER

Section II

Part A

QUESTION ELEVEN (2 marks)

Find the derivative of:

(a) $y = 3x^4 - x + 10$ **1**

....

....

....

....

(b) $y = x\sqrt{x}$ **1**

....

....

....

....

QUESTION TWELVE (4 marks)

Find:

(a) $\displaystyle\int 2x - \frac{2}{x}\,dx$ **2**

....

....

....

....

....

....

(b) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin x\,dx$ **2**

....

....

....

....

....

....

QUESTION THIRTEEN (3 marks)

Calculate the sum of the arithmetic series $10 + 12 + 14 + \cdots + 128$. **3**

QUESTION FOURTEEN (3 marks)

The table shows the probability distribution of a discrete random variable.

x	2	4	10	20
$P(X = x)$	k	0.05	0.35	$3k$

(a) Show that $k = 0.15$. **1**

....

....

....

....

(b) State the mode. **1**

....

....

(c) Find the expected value. **1**

....

....

....

....

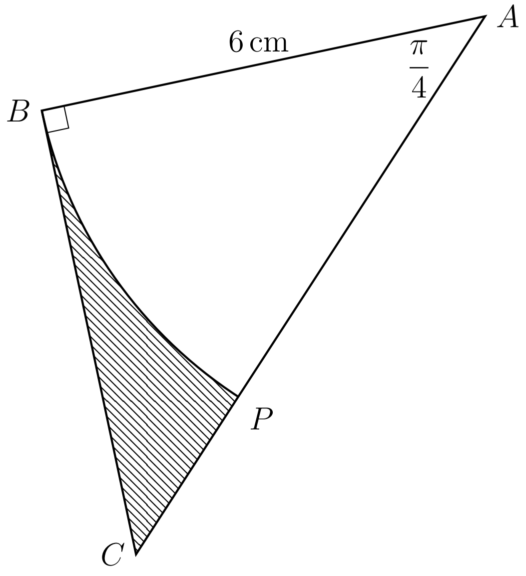
QUESTION FIFTEEN (4 marks)

(a) Find the equation of the tangent to the curve $y = (x+2)(x-2)$ at the point $(2, 0)$. **3**

(b) Find the angle of inclination of the tangent, correct to the nearest degree. **1**

QUESTION SIXTEEN (2 marks)

In the diagram ABC is a triangle with a right angle at B, $AB = 6\,\text{cm}$ and $\angle CAB = \dfrac{\pi}{4}$. Also, PB is an arc of a circle with centre A, radius AB, which meets AC at P.



Find the exact area of the shaded region. **2**

....

....

....

....

....

....

....

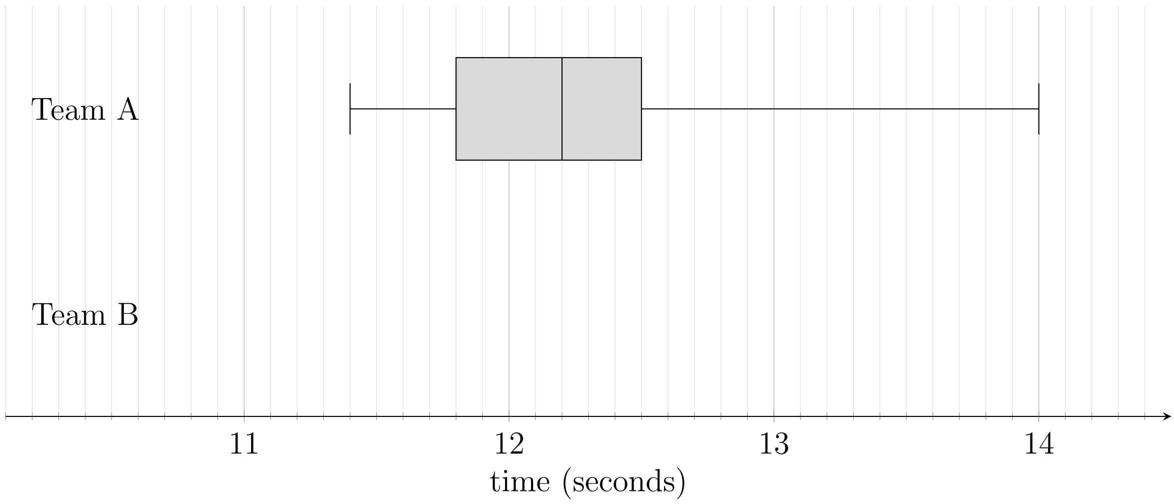
....

....

....

QUESTION SEVENTEEN (4 marks)

Two athletic teams, each with 12 members, participated in a 100 m sprint competition. The times for each person in Team A are different, and the times for each person in Team B are different. The results for Team A are displayed in the box-and-whisker plot below.



(a) The five-number summary for Team B's results is 11.2, 12, 12.5, 13, 13.5. **1**
Draw the box-and-whisker plot to display Team B's results in the space provided above.

(b) State the type of skewness of the distribution of Team A. **1**

....

....

(c) One member from each team is selected at random. Find the probability that at least one of them finished in under 12.5 seconds in the 100 m sprint. **2**

....

....

....

....

....

....

QUESTION EIGHTEEN (2 marks)

Let $f(x) = |x|$ and $g(x) = e^x$.

(a) Find $g\left(f(x)\right)$. **1**

....

....

....

....

....

....

(b) Show that $g\left(f(x)\right)$ is an even function. **1**

....

....

....

....

....

....

CANDIDATE NUMBER

Section II

Part B

QUESTION NINETEEN (3 marks)

(a) Differentiate $y = \sqrt{2x^2 - 5}$. **2**

....

....

....

....

....

....

....

....

(b) Hence, or otherwise, find $\displaystyle\int \frac{x}{\sqrt{2x^2 - 5}}\,dx$. **1**

....

....

....

....

....

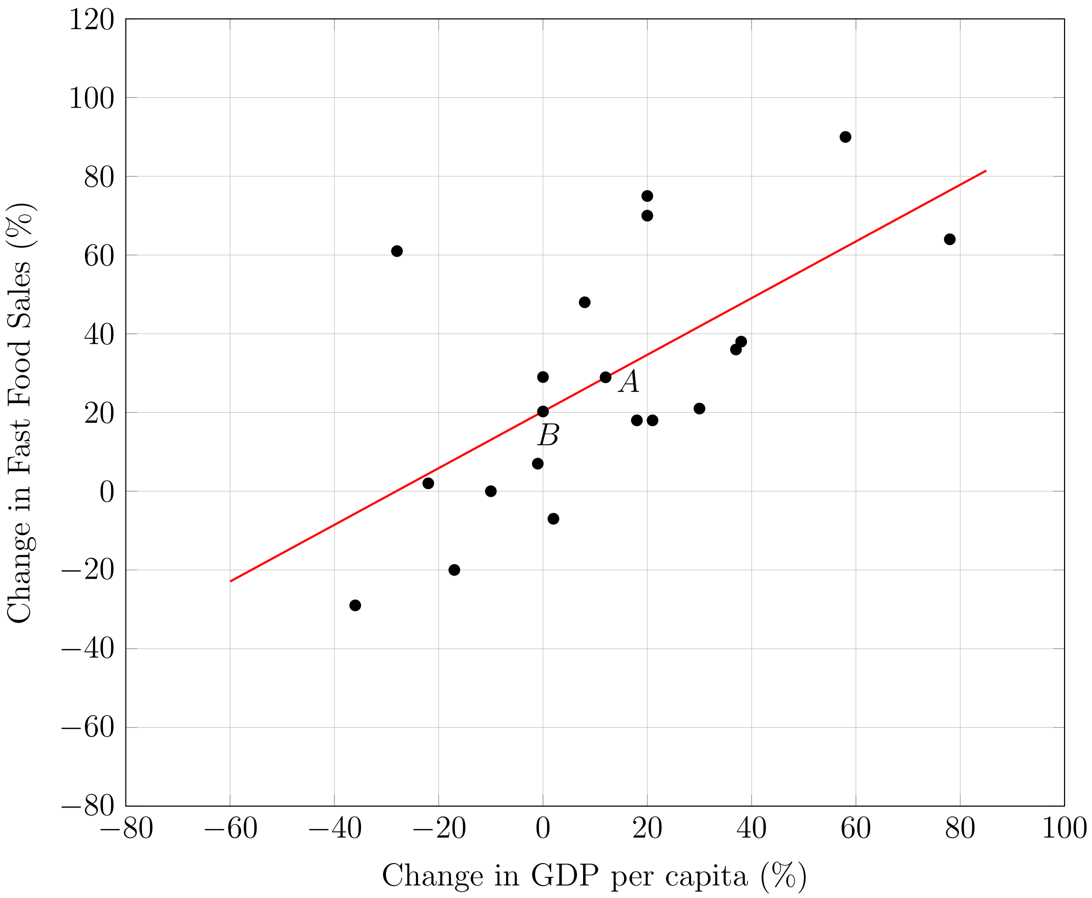
....

....

QUESTION TWENTY (4 marks)

The graph below is adapted from a diagram in the New York Times series “Planet Fate”. It shows how the local fast-food sales change as the country prospers.

The two variables in this scatterplot are the percentage change of gross domestic product (GDP) per capita (horizontal axis, x) and the percentage change in fast food sales from 2010 to 2015 (vertical axis, y).



Using the data provided, the least-squares regression line was found to pass through points $A\,(12,\ 28.94)$ and $B\,(0,\ 20.3)$. The correlation coefficient was calculated to be $r = 0.53$.

(a) Describe the strength and direction of the correlation between the change in GDP per capita and the change in fast food sales. **1**

....

....

....

....

(b) Find the equation of the regression line. **2**

....

....

....

....

....

....

(c) Interpret the meaning of the gradient of the regression line, with reference to the context. **1**

....

....

....

....

QUESTION TWENTY ONE (2 marks)

A curve passing through (0, 4) has gradient function $\dfrac{dy}{dx} = \dfrac{2x}{x^2+1}$. **2**
Find the equation of the curve.

....

....

....

....

....

....

....

....

....

QUESTION TWENTY TWO (5 marks)

Recycled plastic materials can contain bacteria from previous usage. To ensure the plastic is clean and safe for recycling, it is heated to 135°C for a certain period to kill the bacteria present.
The number of bacteria N in the plastic after t minutes at 135°C is given by:

$$N(t) = 4800e^{-0.3t}, \quad t \geq 0$$

(a) Find the initial number of bacteria present in the plastic. **1**

....

....

....

(b) Find the time in minutes to kill 50% of the bacteria. Give your answer correct to the nearest second. **2**

....

....

....

....

....

....

(c) Find the rate of decrease of the bacteria after 5 minutes. Give your answer to the nearest whole number. **2**

....

....

....

....

....

....

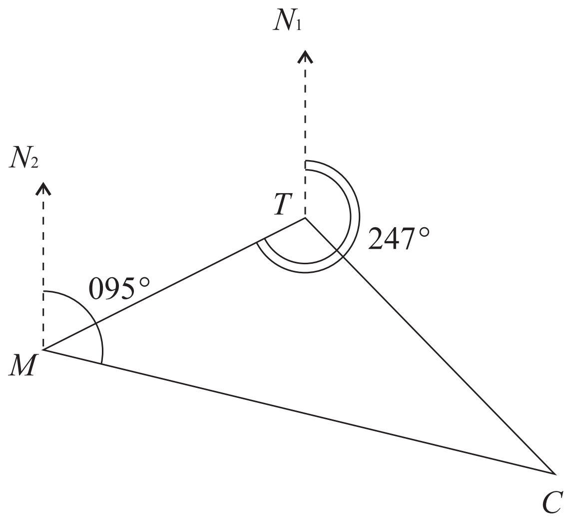
....

....

QUESTION TWENTY THREE (5 marks)

The Carpathia was a passenger steamship renowned for rescuing all the survivors of the Titanic in 1912. While the Carpathia was not the closest ship, once it received the distress signal from the Titanic it swiftly altered its course and reached the scene first.

At the time the Titanic sent the distress signal, another ship, Mount Temple, was 50 nautical miles away, on a bearing of 247°T from the Titanic. The Carpathia was 97 nautical miles from Mount Temple, on a bearing of 095°T. The diagram below illustrates the positions of the Titanic (T), the Carpathia (C), and Mount Temple (M) when the distress signal was sent.



(a) Show that $\angle TMC = 28°$. **1**

....

....

....

....

....

....

....

....

....

....

....

....

(b) Find the distance between the Titanic and the Carpathia when the distress signal was sent. Give your answer to the nearest nautical mile. **2**

....

....

....

....

....

....

....

....

....

....

(c) Hence find the bearing of the Carpathia from the Titanic when the distress signal was sent. Give your answer to the nearest degree. **2**

....

....

....

....

....

....

....

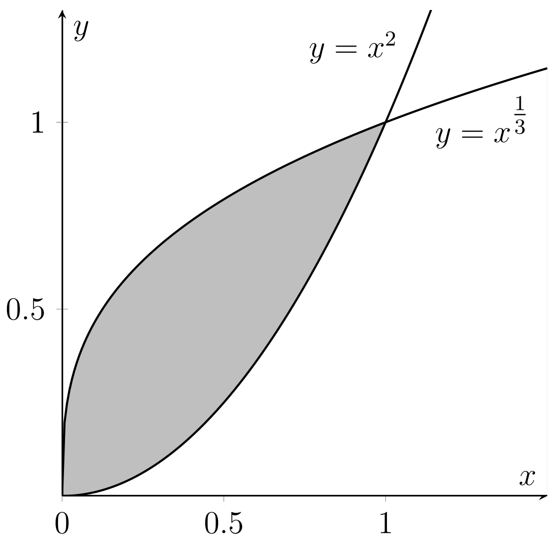
....

....

....

QUESTION TWENTY FOUR (3 marks)

The curves of $y = x^2$ and $y = x^{\frac{1}{3}}$ intersect at the origin and $(1,\ 1)$. Do NOT prove this. **3**



Find the area enclosed by the curves of $y = x^2$ and $y = x^{\frac{1}{3}}$, shown as the shaded region in the diagram.

CANDIDATE NUMBER

Section II

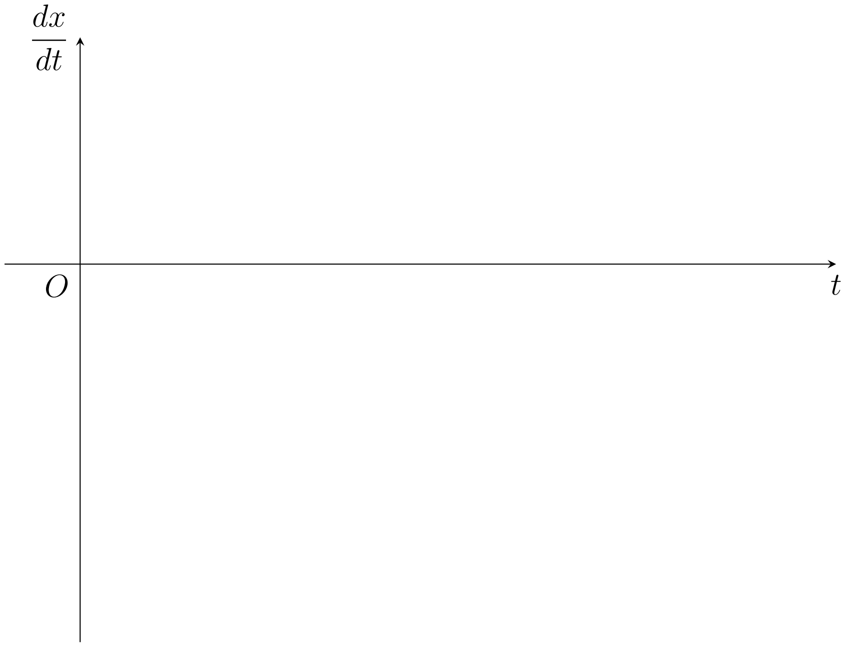
Part C

QUESTION TWENTY FIVE (5 marks)

A particle moves in a straight line with acceleration $\dfrac{d^2x}{dt^2} = 1 - 2t$, for $0 \leq t \leq 2$.
The particle was initially at rest at the origin.

(a) Determine $\dfrac{dx}{dt}$ as a function of t. **1**

(b) Sketch the graph of $\dfrac{dx}{dt}$ for $0 \leq t \leq 2$ below, showing any stationary points, end points and intercepts with the axes. **2**



(c) Find the distance travelled over the first two seconds. **2**

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

QUESTION TWENTY SIX (2 marks)

Consider the geometric series: $\sin^2\theta + \sin^2\theta\cos^2\theta + \sin^2\theta\cos^4\theta + \cdots$, where $0 < \theta < \dfrac{\pi}{2}$.

(a) Show that the limiting sum of the series exists for the given domain. **1**

....

....

....

....

....

....

....

....

(b) Find the limiting sum of the series. **1**

....

....

....

....

....

....

QUESTION TWENTY SEVEN (5 marks)

(a) On the same number plane, sketch the graphs of the functions $f(x) = 3 - x$ and $g(x) = |2x - 4|$, showing any intercepts and points of intersection. You may use the working out space provided below. **3**

(b) Hence, or otherwise, solve the inequality $|2x - 4| < 3 - x$. **1**

....

....

....

....

....

....

....

....

....

(c) Find all the possible values of the constant a so that $|2x - 4| < a - x$ has real solutions. **1**

....

....

....

....

....

....

....

....

QUESTION TWENTY EIGHT (6 marks)

The moon is illuminated by reflected sunlight. The proportion of the moon which is illuminated each night can be approximately modelled by the function below:

$$M(t) = 0.5\sin\left(\frac{\pi}{15}(t-13)\right) + 0.5,$$

where t is the time in days after midnight on the 1st of August.

(a) Find the proportion of the moon which is illuminated at midnight on the following days:

(i) 14th August (that is, $t = 13$) **1**

....

....

....

(ii) 29th August (that is, $t = 28$) **1**

....

....

....

(b) Show that the time period between consecutive full moons is 30 days, according to the model provided here. **1**

....

....

....

(c) For $0 \leq t \leq 30$, find the values of t when the moon is not illuminated at all. **1**

....

....

....

....

....

....

....

....

(d) Sketch the graph of $M(t)$ for $0 \leq t \leq 30$, showing any stationary points, points of inflection and intercepts with the axes. **2**

....

....

....

....

....

....

....

....

....

....

QUESTION TWENTY NINE (5 marks)

Let $f(x) = xe^{-x^2}$.

(a) Find the coordinates of the stationary points of $y = f(x)$. You do NOT need to check the nature of the stationary points. **3**

(b) Without using any further calculus, sketch the graph of $y = f(x)$, showing stationary points, any intercepts and asymptotes. **2**

CANDIDATE NUMBER

Section II

Part D

QUESTION THIRTY (9 marks)

Jackie took a loan of \$500 000, which is to be repaid over 30 years, by 360 equal monthly repayments of \$$M$. The first repayment will be made one month after the loan is taken out. Interest on the loan is charged at 6% per annum, compounded monthly. Let A_n be the amount owing after the nth monthly repayment.

(a) Show that $A_2 = 500\,000(1.005)^2 - M(1 + 1.005)$. **2**

....

....

....

....

....

....

(b) Show that $A_n = 500\,000(1.005)^n - M\left(\dfrac{(1.005)^n - 1}{0.005}\right)$. **1**

....

....

....

....

....

....

....

....

(c) Show that the monthly repayment is \$2998, correct to the nearest dollar. **2**

(d) Immediately after the 120th repayment, Jackie made a one-off payment, reducing the amount owing to \$350 000. The interest rate stays at 6% per annum, compounded monthly, and her monthly repayment remains \$2998.

(i) After how many more months will the amount owing be completely repaid? **2**

(ii) Find the amount of the last repayment. Give your answer correct to the nearest dollar. **2**

....

....

....

....

....

....

....

....

....

....

....

....

....

....

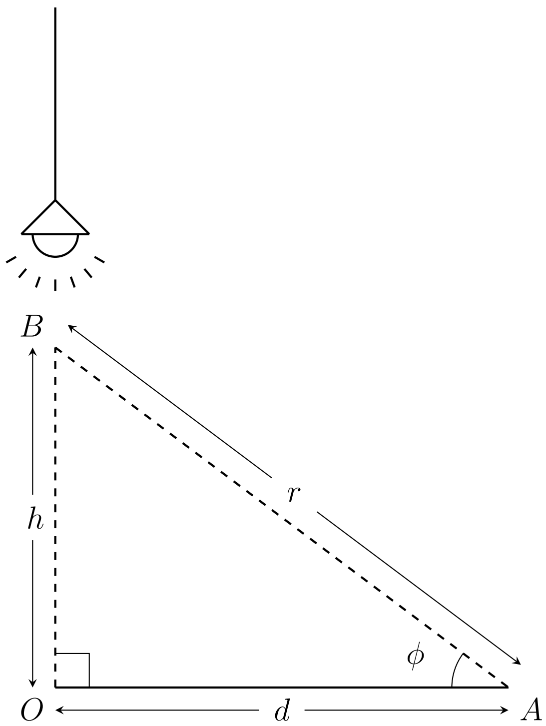
....

QUESTION THIRTY ONE (5 marks)

The lamp shown in the diagram below can move along the vertical line OB using a pulley system. The point A lies on the horizontal line OA at a distance of d from O. The intensity I of light from the lamp at point A is inversely proportional to the square of its distance r from the lamp. The intensity I is also directly proportional to $\sin\phi$, where ϕ is the angle of elevation from A to the lamp. Hence, the formula for I can be expressed as: **5**

$$I = \frac{C\sin\phi}{r^2}, \quad \text{where } C \text{ is a positive constant.}$$

Determine the height h of the lamp above the line OA that maximizes the intensity of light at the point A. Express your answer in terms of d.



....

....

....

....

....

....

....

....

QUESTION THIRTY TWO (7 marks)

Consider the functions $f(x) = 2^x$ and $g(x) = 3^x$.

(a) Find a single transformation that transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. **2**

....

....

....

....

....

....

(b) A new function $h(x)$ is found by taking the graph of $y = -g(-x)$ and translating it up by b units, where $b > 1$. **2**
Sketch the graph of $y = h(x)$ showing the x-intercept and the asymptote in terms of b.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(c) Hence find the value of b such that the area bounded by the curve $y = h(x)$, the x-axis and the y-axis is exactly $\dfrac{1}{\ln 3}$ square units. **3**

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

END OF PAPER

Section I

Questions in this section are multiple-choice.
Record the single best answer for each question on the provided answer sheet.

---

1. What is the domain of the function $y = \dfrac{1}{\sqrt{x}}$? $x>0$

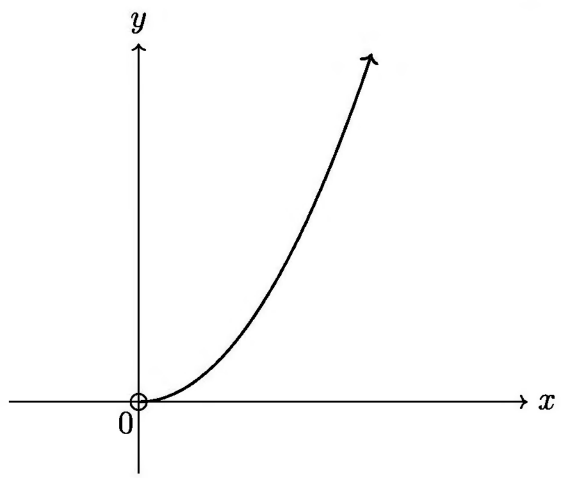
   (A) $(-\infty, 0)$

   (B) $(-\infty, 0]$

   (C) $[0, \infty)$

   (D) $(0, \infty)$

2. Which equation best represents the graph drawn below?



   (A) $y = 4x$, $x > 0$

   (B) $y = 4^x$, $x > 0$

   (C) $y = 4x^2$, $x > 0$

   (D) $y = \dfrac{4}{x}$, $x > 0$

3. A table of future value interest factors is shown below.

Table of future value interest factors

Period	*Interest rate per period*					
	1%	2%	3%	4%	6%	8%
2	2.0100	2.0200	2.0300	2.0400	2.0600	2.0800
3	3.0301	3.0604	3.0909	3.1216	3.1836	3.2464
4	4.0604	4.1216	4.1836	4.2465	4.3746	4.5061
5	5.1010	5.2040	5.3091	5.4163	5.6371	5.8666
6	6.1520	6.3081	6.4684	6.6330	6.9753	7.3359
7	7.2135	7.4343	7.6625	7.8983	8.3938	8.9228
8	8.2857	8.5830	8.8923	9.2142	9.8975	10.6366
9	9.3685	9.7546	10.1591	10.5828	11.4913	12.4876
10	10.4622	10.9497	11.4639	12.0061	13.1808	14.4866
11	11.5668	12.1687	12.8078	13.4864	14.9716	16.6455
12	12.6825	13.4121	14.1920	15.0258	16.8699	18.9771

An annuity involves making equal contributions of \$2500 into an account every quarter for 2 years at an interest rate of 4% per annum.
Based on the information provided, what is the future value of this annuity?

(A) \$5100

(B) \$20 714.25

(C) \$23 035.50

(D) \$26 591.50

8 quarters.

1% per quarter

4. What is the derivative of the function $f(x) = \log_e x^3$, with respect to x?

(A) $f'(x) = \dfrac{1}{3x}$

(B) $f'(x) = \dfrac{3}{x}$

(C) $f'(x) = \left(\dfrac{1}{x}\right)^3$

(D) $f'(x) = 3x$

$f(x) = 3 \ln x$

$f'(x) = \frac{3}{x}$

5. Consider a dataset with mean 10 and standard deviation 2. Two scores, 10.5 and 9.5 are added to the dataset. Which choice shows how the mean and standard deviation are affected by the added scores?

   (A) Both mean and standard deviation will change.

   (B) Only standard deviation will decrease.

   (C) Only standard deviation will increase.

   (D) Both mean and standard deviation will stay the same.

6. Researchers are investigating population density, house size and the distance people live from the centre of a city. Data is plotted with distance on the horizontal axis.

   - For the graph of population density vs distance, the Pearson's correlation coefficient is $r = -0.563$.
   - For house size vs distance, the Pearson's correlation coefficient is $r = 0.357$.

   Given this information, which one of the following statements must be true?

   (A) Population density tends to be higher when it is farther away from the city centre.

   (B) House size tends to be larger when it is closer to the city centre.

   (C) House size is positively correlated with population density.

   (D) Population density is more strongly correlated with distance from the city centre than house size.

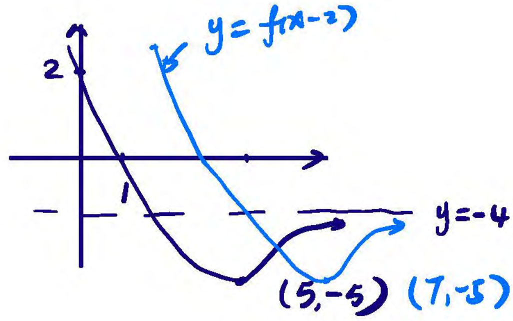
7. Suppose the graph of $y = f(x)$ has an x-intercept at 1, a y-intercept at 2, a minimum turning point $(5, -5)$ and a single horizontal asymptote $y = -4$.
   What can you say is definitely true about the graph of $y = -f(x-2)$?

   (A) Its x-intercept is -1.

   (B) Its y-intercept is -4.

   (C) It has a maximum turning point at $(7, 5)$.

   (D) Its horizontal asymptote cannot be determined.



8. A bag contains six white socks and eight black socks. Two socks are selected at random without replacement. Given the two selected socks are the same colour, which of the following is the closest to the probability that they are both black?

(A) 31%

(B) 54%

(C) 57%

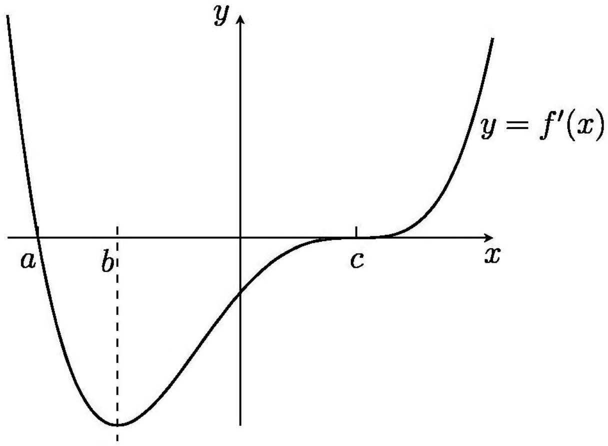
(D) 65%

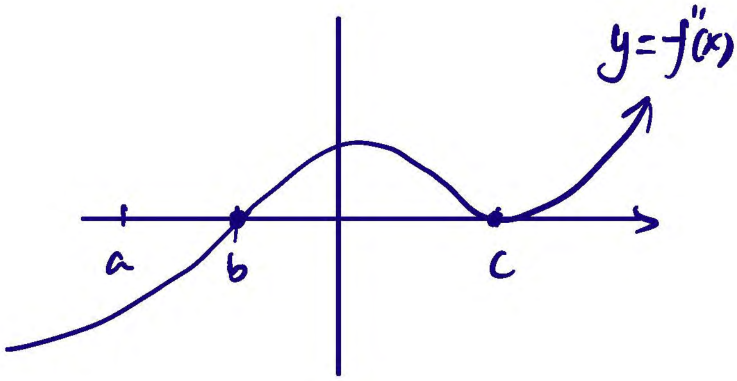
P(Both Black | Same colour)

$= \frac{P(\text{"Both B" and "Same colour"})}{P(\text{Same colour})}$

$= \frac{\frac{8}{14} \times \frac{7}{13}}{\frac{8}{14} \times \frac{7}{13} + \frac{6}{14} \times \frac{5}{13}}$

9. The graph of $y = f'(x)$ is drawn below.





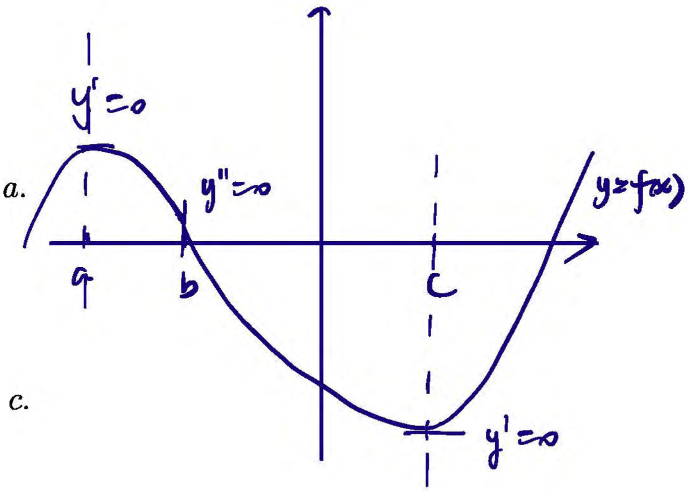
Which of the following statements is true?

(A) $f(x)$ has a minimum turning point at $x = a$.

(B) $f(x)$ has a point of inflection at $x = b$.

(C) $f(x)$ has a maximum turning point at $x = c$.

(D) $f(x)$ has a point of inflection at $x = c$.



10. The function $f(x) = \sqrt{3}\cos(\alpha - 2x)$ is odd. Its graph also increases in the interval of $0 < x < \frac{\pi}{4}$. What is a possible value of α?

(A) $-\frac{\pi}{2}$

(B) 0

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

Odd: $f(0) = \sqrt{3}\cos(\alpha) = 0$

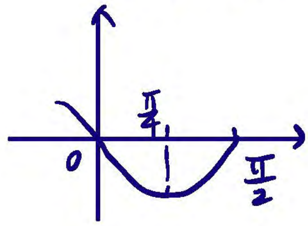
$\cos(\alpha) = 0$

$\therefore$ Eliminate (B) & (C)

If $\alpha = -\frac{\pi}{2}$ $\quad f(x) = \sqrt{3}\cos\left(-\frac{\pi}{2} - 2x\right)$

$= \sqrt{3}\cos\left(\frac{\pi}{2} + 2x\right)$

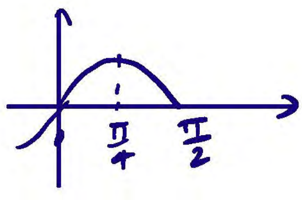
$= -\sqrt{3}\sin 2x$



End of Section I

If $\alpha = \frac{\pi}{2}$, $\quad f(x) = \sqrt{3}\cos\left(\frac{\pi}{2} - 2x\right)$

$= \sqrt{3}\sin 2x$



The paper continues in the next section

CANDIDATE NUMBER

Section II

Part A

QUESTION ELEVEN (2 marks)

Find the derivative of:

(a) $y = 3x^4 - x + 10$ **1**

$y' = 12x^3 - 1$ ✓

(b) $y = x\sqrt{x}$ **1**

$y = x^{\frac{3}{2}}$

$y' = \frac{3}{2}x^{\frac{1}{2}}$ (OR $y' = \frac{3}{2}\sqrt{x}$) ✓

QUESTION TWELVE (4 marks)

Find:

(a) $\displaystyle\int 2x - \frac{2}{x}\, dx$ **2**

$\displaystyle\int 2x - 2x^{-1}\, dx$

$= x^2 - 2\ln|x| + C$ ✓ x^2 OR $\ln|x|$

✓ correct answer.

(b) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin x\, dx$ **2**

$= \left[-\cos x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ ✓ correct primitive

$= -\cos\frac{\pi}{3} + \cos\frac{\pi}{6}$

$= -\frac{1}{2} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}-1}{2}$ ✓

QUESTION THIRTEEN (3 marks)

Calculate the sum of the arithmetic series $10 + 12 + 14 + \cdots + 128$. **3**

$a = 10 \quad d = 2$

$T_n = (n-1)d + a = 128$

$2(n-1) + 10 = 128$

$2(n-1) = 118$

$n - 1 = 59$

$n = 60$ ✓

$S_{60} = \frac{n}{2}(a + l)$

$= \frac{60}{2}(10 + 128)$ ✓

$= 4140$ ✓

QUESTION FOURTEEN (3 marks)

The table shows the probability distribution of a discrete random variable.

x	2	4	10	20
$P(X = x)$	k	0.05	0.35	$3k$

(a) Show that $k = 0.15$. **1**

$k + 0.05 + 0.35 + 3k = 1$
$4k = 0.6$
$k = 0.15$ ✓

(b) State the mode. **1**

Mode = 20 ✓

(c) Find the expected value. **1**

$E(x) = 2 \times 0.15 + 4 \times 0.05 + 10 \times 0.35 + 20 \times 0.45$
$= 13$ ✓

QUESTION FIFTEEN (4 marks)

(a) Find the equation of the tangent to the curve $y = (x + 2)(x - 2)$ at the point $(2, 0)$. **3**

$y = x^2 - 4$

$y' = 2x$ ✓

At $(2, 0)$

$m = 2 \times 2 = 4$ ✓

The tangent:

$y = 4(x - 2)$ ✓

$y = 4x - 8$

(b) Find the angle of inclination of the tangent, correct to the nearest degree. **1**

$m = \tan\alpha = 4$

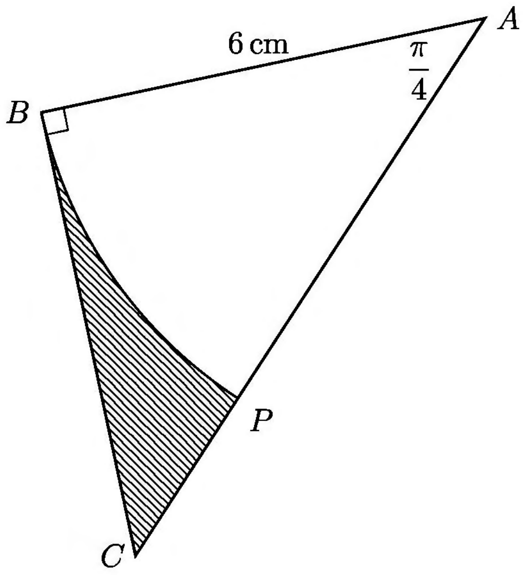
$\alpha = \tan^{-1}(4)$

$= 75.9\ldots$

$\approx 76°$ ✓

QUESTION SIXTEEN (2 marks)

In the diagram ABC is a triangle with a right angle at B, $AB = 6\,\text{cm}$ and $\angle CAB = \dfrac{\pi}{4}$. Also, PB is an arc of a circle with centre A, radius AB, which meets AC at P.



Find the exact area of the shaded region. **2**

$A_{\text{sector}} = \frac{1}{2}\theta r^2$

$= \frac{1}{2}\left(\frac{\pi}{4}\right)(6^2)$

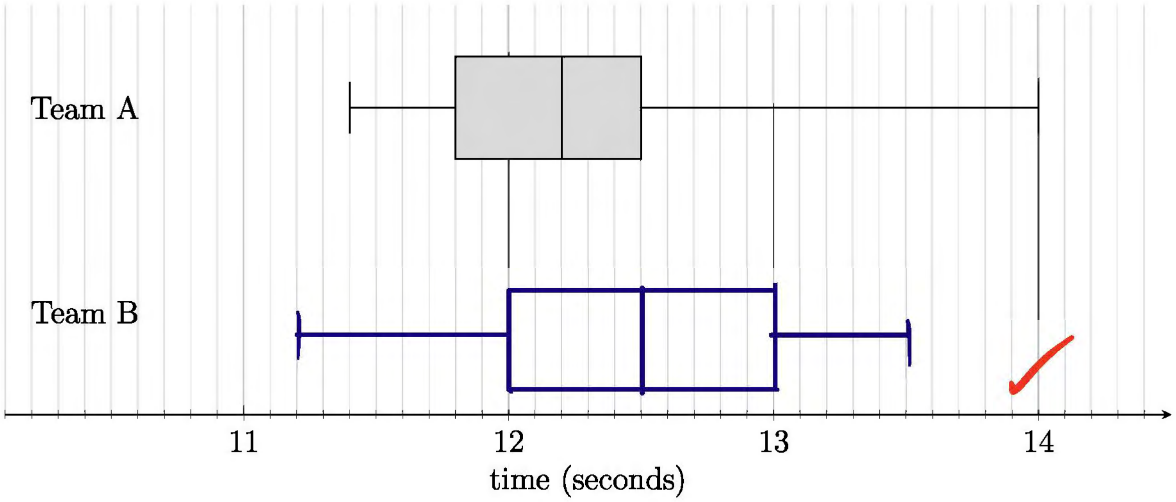
$= \frac{9}{2}\pi \text{ cm}^2$ ✓

$A_{\text{shaded}} = \frac{1}{2}(6)(6) - \frac{9}{2}\pi$

$= 18 - \frac{9}{2}\pi \text{ cm}^2$ ✓

QUESTION SEVENTEEN (4 marks)

Two athletic teams, each with 12 members, participated in a 100 m sprint competition. The times for each person in Team A are different, and the times for each person in Team B are different. The results for Team A are displayed in the box-and-whisker plot below.



(a) The five-number summary for Team B's results is 11.2, 12, 12.5, 13, 13.5. Draw the box-and-whisker plot to display Team B's results in the space provided above. **1**

(b) State the type of skewness of the distribution of Team A. **1**

Positively skewed.

(c) One member from each team is selected at random. Find the probability that at least one of them finished in under 12.5 seconds in the 100 m sprint. **2**

P(both spent more than 12.5) = $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

P(at least one of them spent less than 12.5s) = $1 - \frac{1}{8}$

$= \frac{7}{8}$

QUESTION EIGHTEEN (2 marks)

Let $f(x) = |x|$ and $g(x) = e^x$.

(a) Find $g(f(x))$. **1**

$g(f(x)) = g(|x|) = e^{|x|}$ ✓

(b) Show that $g(f(x))$ is an even function. **1**

$g(f(-x)) = e^{|-x|} = e^{|x|} = g(f(x))$ ✓

$\therefore$ It is even.

CANDIDATE NUMBER

Section II

Part B

QUESTION NINETEEN (3 marks)

(a) Differentiate $y = \sqrt{2x^2 - 5}$. **2**

$y = (2x^2 - 5)^{\frac{1}{2}}$

$y' = \frac{1}{2}(4x)(2x^2 - 5)^{-\frac{1}{2}}$ ✓ for $\frac{1}{2}(2x^2-5)^{-\frac{1}{2}}$

$= \dfrac{2x}{\sqrt{2x^2 - 5}}$ ✓

(b) Hence, or otherwise, find $\displaystyle\int \frac{x}{\sqrt{2x^2 - 5}}\,dx$. **1**

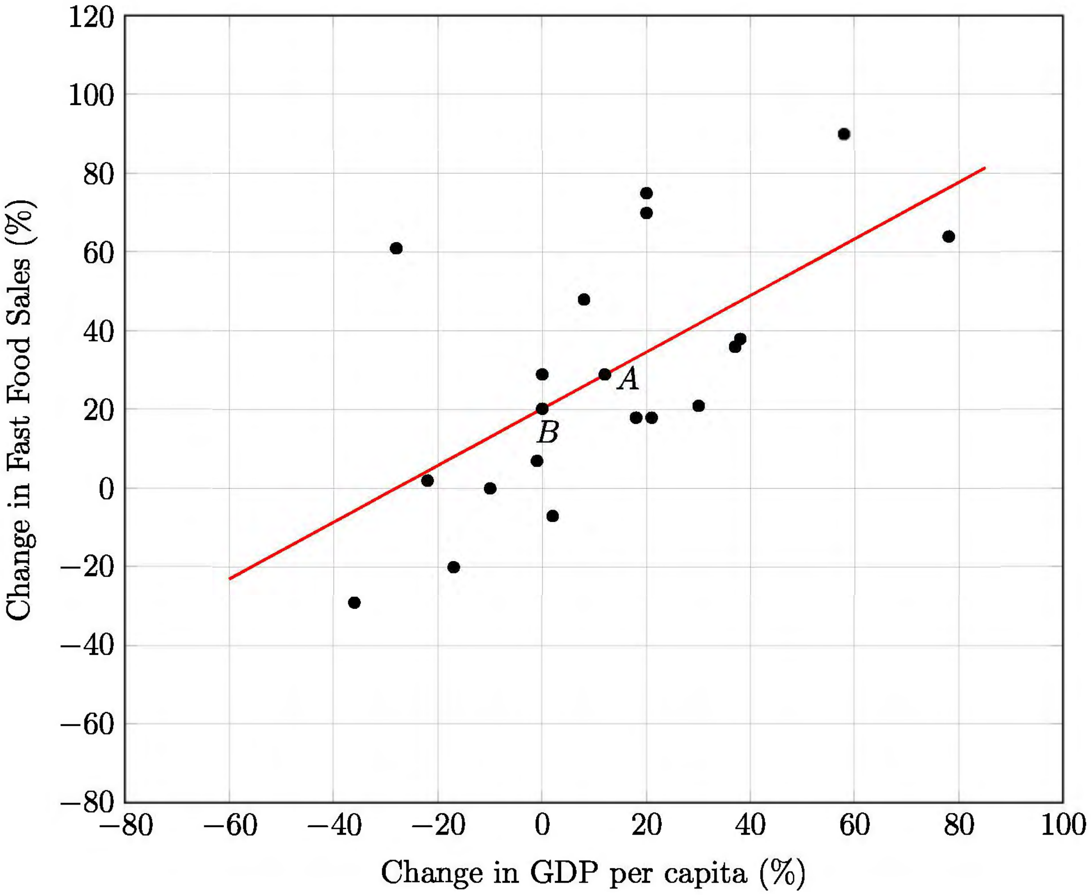
$\frac{1}{2}\displaystyle\int \frac{2x}{\sqrt{2x^2-5}}\,dx = \frac{\sqrt{2x^2-5}}{2} + C$ ✓

(Award even if "C" is missing)

QUESTION TWENTY (4 marks)

The graph below is adapted from a diagram in the New York Times series "Planet Fate". It shows how the local fast-food sales change as the country prospers.

The two variables in this scatterplot are the percentage change of gross domestic product (GDP) per capita (horizontal axis, x) and the percentage change in fast food sales from 2010 to 2015 (vertical axis, y).



Using the data provided, the least-squares regression line was found to pass through points $A(12,\ 28.94)$ and $B(0,\ 20.3)$. The correlation coefficient was calculated to be $r = 0.53$.

(a) Describe the strength and direction of the correlation between the change in GDP per capita and the change in fast food sales. **1**

It's a moderate positive correlation ✓ (Both)

(b) Find the equation of the regression line. **2**

$m = \frac{28.94 - 20.3}{12 - 0} = 0.72$ ✓

$y = 0.72x + 20.3$ ✓

(c) Interpret the meaning of the gradient of the regression line, with reference to the context. **1**

For each percent increase in GDP, there was a 0.72% increase in fast food sales on average. ✓

[Must interpret meaning of $m = 0.72$, describing the rate of change.]

QUESTION TWENTY ONE (2 marks)

A curve passing through (0, 4) has gradient function $\dfrac{dy}{dx} = \dfrac{2x}{x^2+1}$. **2**
Find the equation of the curve.

$y = \int \frac{2x}{x^2+1}\,dx$

$= \ln(x^2+1) + c$ ✓ integrating

When $x = 0$, $y = 4$

$\ln(1) + c = 4$

$c = 4$

$\therefore y = \ln(x^2+1) + 4$ ✓ equation of curve.

QUESTION TWENTY TWO (5 marks)

Recycled plastic materials can contain bacteria from previous usage. To ensure the plastic is clean and safe for recycling, it is heated to 135°C for a certain period to kill the bacteria present.
The number of bacteria N in the plastic after t minutes at 135°C is given by:

$$N(t) = 4800e^{-0.3t}, \quad t \geq 0$$

(a) Find the initial number of bacteria present in the plastic. **1**

$N(0) = 4800e^0 = 4800$ bacteria ✓

(b) Find the time in minutes to kill 50% of the bacteria. Give your answer correct to the nearest second. **2**

$N(t) = 50\% \times 4800 = 2400$

$4800e^{-0.3t} = 2400$

$e^{-0.3t} = 0.5$ ✓

$-0.3t = \ln(\frac{1}{2})$

$t = -\frac{\ln(0.5)}{0.3}$

$t \approx 2.310\ldots$

$t \approx$ 2 minutes 19 seconds. ✓

(c) Find the rate of decrease of the bacteria after 5 minutes. Give your answer to the nearest whole number. **2**

$\frac{dN}{dt} = 4800(-0.3)e^{-0.3t}$

$= -1440e^{-0.3t}$ ✓

When $t = 5$ min

$\frac{dN}{dt} = -1440 \times e^{-1.5}$

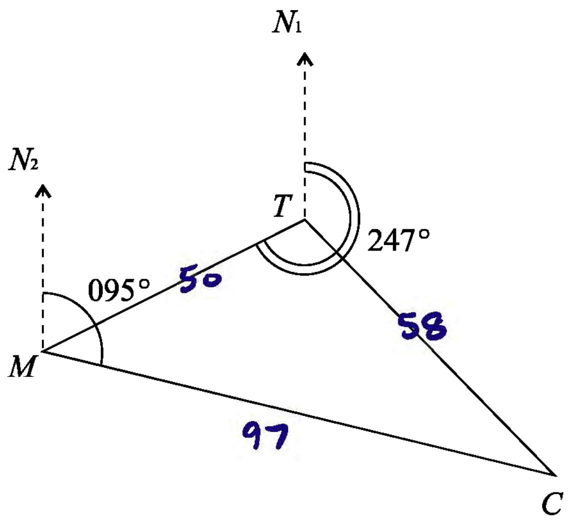
$= -321$ bacteria / min ✓

The number decreases at 321 bacteria/min

QUESTION TWENTY THREE (5 marks)

The Carpathia was a passenger steamship renowned for rescuing all the survivors of the Titanic in 1912. While the Carpathia was not the closest ship, once it received the distress signal from the Titanic it swiftly altered its course and reached the scene first.

At the time the Titanic sent the distress signal, another ship, Mount Temple, was 50 nautical miles away, on a bearing of 247°T from the Titanic. The Carpathia was 97 nautical miles from Mount Temple, on a bearing of 095°T. The diagram below illustrates the positions of the Titanic (T), the Carpathia (C), and Mount Temple (M) when the distress signal was sent.



(a) Show that $\angle TMC = 28°$. **1**

$\angle MTN_1 = 360° - 247° = 113°$

$\angle N_2MT = 180° - 113° = 67°$ ($MN_2 \parallel TN_1$, co-interior angles)

$\angle TMC = 95° - 67° = 28°$ (Adjacent angles)

✓

(b) Find the distance between the Titanic and the Carpathia when the distress signal was sent. Give your answer to the nearest nautical mile. **2**

In $\triangle MTC$

$TC^2 = MT^2 + MC^2 - 2\,MT \times MC \times \cos 28^\circ$

$TC^2 = 50^2 + 97^2 - 2(50)(97)\cos 28^\circ$ ✓

$TC = \sqrt{50^2 + 97^2 - 9700\cos 28^\circ}$

$\doteqdot 58$ nautical miles. ✓

(c) Hence find the bearing of the Carpathia from the Titanic when the distress signal was sent. Give your answer to the nearest degree. **2**

$\dfrac{\sin\angle MTC}{97} = \dfrac{\sin 28^\circ}{58}$

$\sin\angle MTC = \dfrac{97}{58}\sin 28^\circ$

$\angle MTC \doteqdot 51.73^\circ$ OR $180^\circ - 51.73^\circ \doteqdot 128.27^\circ$

$\because$ MC is the longest side $\therefore \angle MTC \doteqdot 128.27^\circ$ ✓

(So that $\angle TCM$ is acute)

Must justify

$247^\circ - 128.27^\circ = 118.73^\circ \approx 119^\circ T$ ✓

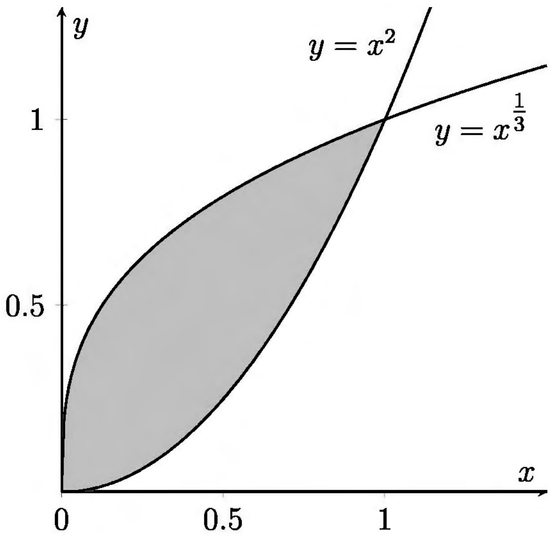
$\therefore$ The bearing of C from T is 119° ~~true~~ bearing.

Method 2: By cosine rule:

$\cos\angle MTC = \dfrac{50^2 + 58^2 - 97^2}{2(50)(58)}$

QUESTION TWENTY FOUR (3 marks)

The curves of $y = x^2$ and $y = x^{\frac{1}{3}}$ intersect at the origin and $(1,\ 1)$. Do NOT prove this. **3**



Find the area enclosed by the curves of $y = x^2$ and $y = x^{\frac{1}{3}}$, shown as the shaded region in the diagram.

$\int_0^1 x^{\frac{1}{3}} - x^2 \, dx$ ✓ a correct expression for the area.

$= \left[\frac{1}{1+\frac{1}{3}} x^{\frac{4}{3}} - \frac{1}{3} x^3 \right]_0^1$

$= \left[\frac{3}{4} x^{\frac{4}{3}} - \frac{1}{3} x^3 \right]_0^1$ ✓ finding primitive

$= \left(\frac{3}{4} - \frac{1}{3} \right) - (0 - 0)$

$= \frac{5}{12}$ unit^2 ✓

CANDIDATE NUMBER

Section II

Part C

QUESTION TWENTY FIVE (5 marks)

A particle moves in a straight line with acceleration $\dfrac{d^2x}{dt^2} = 1 - 2t$, for $0 \le t \le 2$.
The particle was initially at rest at the origin.

(a) Determine $\dfrac{dx}{dt}$ as a function of t. **1**

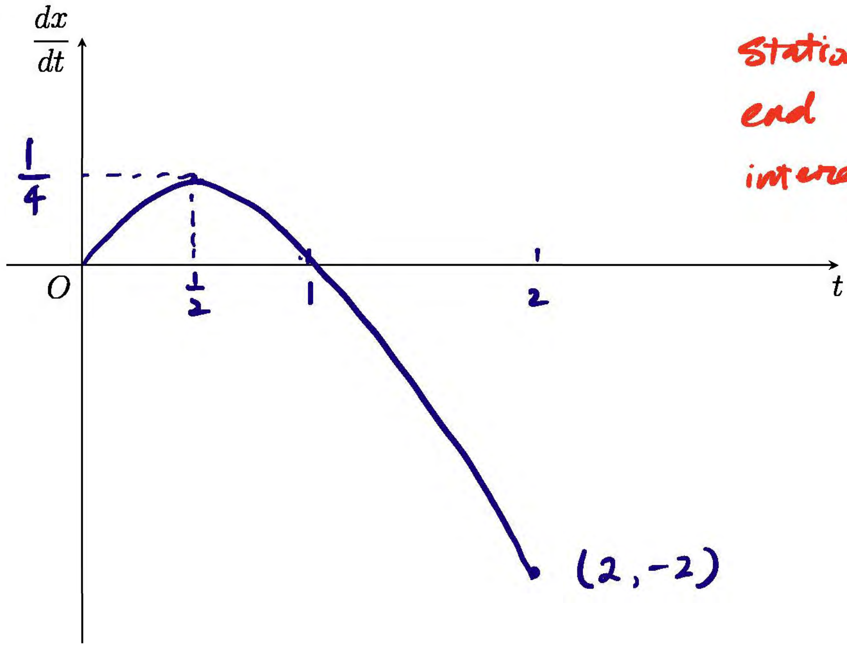
$\dfrac{dx}{dt} = \int 1-2t \, dt$

$= t - t^2 + C$

When $t=0$ $\quad \dfrac{dx}{dt} = C = 0$

$\therefore \dfrac{dx}{dt} = t - t^2$ ✓

(b) Sketch the graph of $\dfrac{dx}{dt}$ for $0 \le t \le 2$ below, showing any stationary points, end points and intercepts with the axes. **2**



Stationary pt. ✓ for 1
end pts.
intercepts ✓✓ for all.

(c) Find the distance travelled over the first two seconds. **2**

$x_1 = \int_0^1 t - t^2 \, dt$

$= \left[\frac{t^2}{2} - \frac{t^3}{3} \right]_0^1$

$= \frac{1}{6}$ ✓

$x_2 = \int_1^2 t - t^2 \, dt$

$= \left[\frac{t^2}{2} - \frac{t^3}{3} \right]_1^2$

$= \left(\frac{4}{2} - \frac{8}{3} \right) - \left(\frac{1}{2} - \frac{1}{3} \right)$

$= -\frac{5}{6}$

$\therefore$ The total distance $= \frac{1}{6} + \left| -\frac{5}{6} \right| = 1$ ✓

QUESTION TWENTY SIX (2 marks)

Consider the geometric series: $\sin^2\theta + \sin^2\theta\cos^2\theta + \sin^2\theta\cos^4\theta + \cdots$, where $0 < \theta < \frac{\pi}{2}$.

(a) Show that the limiting sum of the series exists for the given domain. **1**

The common ratio $= \cos^2\theta$

$0 < \theta < \frac{\pi}{2}$, $0 < \cos\theta < 1$

$0 < \cos^2\theta < 1$

$\therefore |r| < 1$

$\therefore$ The limiting sum exists.

✓

(b) Find the limiting sum of the series. **1**

$S = \frac{a}{1-r}$

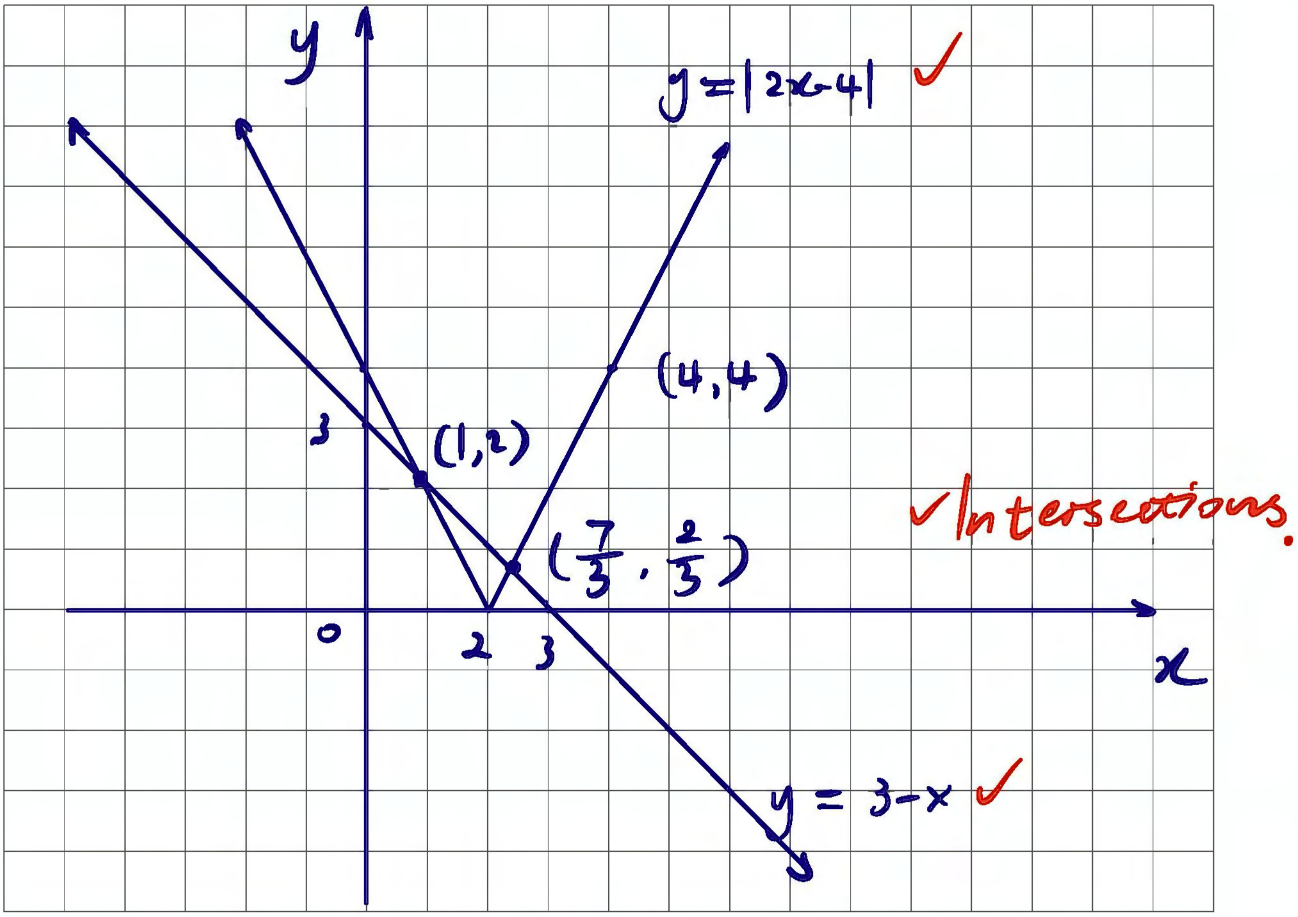
$= \frac{\sin^2\theta}{1-\cos^2\theta}$

$= \frac{\sin^2\theta}{\sin^2\theta}$

$= 1$ ✓

QUESTION TWENTY SEVEN (5 marks)

(a) On the same number plane, sketch the graphs of the functions $f(x) = 3 - x$ and $g(x) = |2x - 4|$, showing any intercepts and points of intersection. You may use the working out space provided below. **3**



When $x > 2$. When $x < 2$.

$\begin{cases} y = 3-x \\ y = 2x-4 \end{cases}$ $\begin{cases} y = 3-x \\ y = 4-2x \end{cases}$

$3-x = 2x-4$ $3-x = 4-2x$

$7 = 3x$ $x = 1$

$x = \frac{7}{3}$ $y = 2$

$x=\frac{7}{3}$, $y = \frac{2}{3}$

(b) Hence, or otherwise, solve the inequality $|2x-4| < 3-x$. **1**

$\therefore \quad 1 < x < \frac{7}{3}$ ✓

(c) Find all the possible values of the constant a so that $|2x-4| < a-x$ has real solutions. **1**

$|2x-4| < a-x$ has solutions as long as $y=a-x$ and $y=|2x-4|$ intersect before the cusp.

Suppose $y=a-x$ passes through $(2,0)$

$0 = a-2$

$a = 2$

$\therefore \quad a > 2$. ✓

QUESTION TWENTY EIGHT (6 marks)

The moon is illuminated by reflected sunlight. The proportion of the moon which is illuminated each night can be approximately modelled by the function below:

$$M(t) = 0.5\sin\left(\frac{\pi}{15}(t-13)\right) + 0.5,$$

where t is the time in days after midnight on the 1st of August.

(a) Find the proportion of the moon which is illuminated at midnight on the following days:

(i) 14th August (that is, $t = 13$) **1**

$t = 13$. $M(13) = 0.5\sin 0 + 0.5$

$= 0.5$ ✓

(ii) 29th August (that is, $t = 28$) **1**

$t = 28$ $M(28) = 0.5\sin\left(\frac{\pi}{15} \times 15\right) + 0.5$

$= 0.5\sin\pi + 0.5$

$= 0.5$ ✓

(b) Show that the time period between consecutive full moons is 30 days, according to the model provided here. **1**

$T = \dfrac{2\pi}{\frac{\pi}{15}} = 30$ days. ✓

(c) For $0 \le t \le 30$, find the values of t when the moon is not illuminated at all. **1**

$M(t) = 0$

$0.5 \sin\left(\frac{\pi}{15}(t-13)\right) + 0.5 = 0$

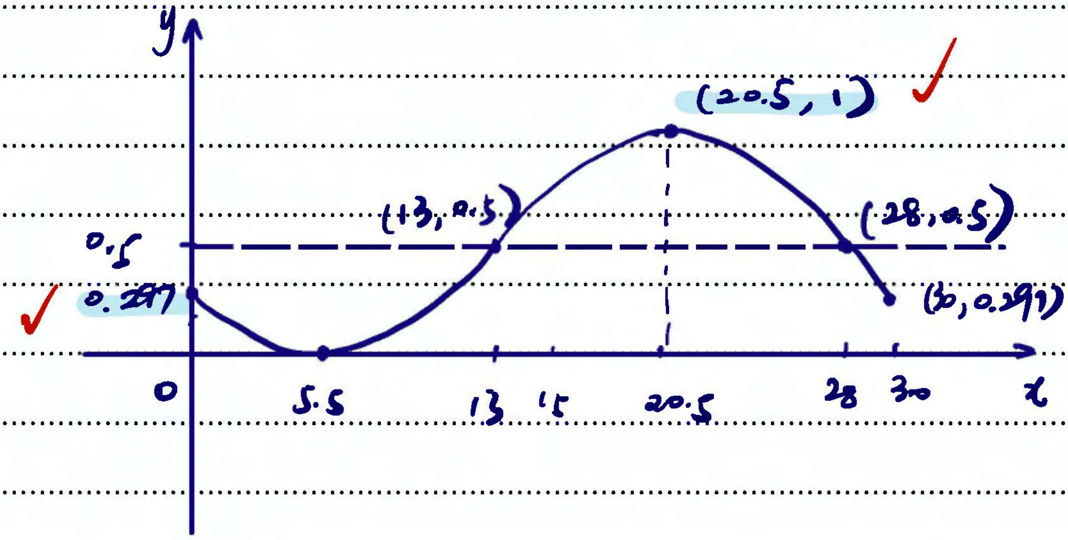
$\sin\left(\frac{\pi}{15}(t-13)\right) = -1, \quad -\frac{13\pi}{15} \le \frac{\pi}{15}(t-13) \le \frac{17}{15}\pi$

$\frac{\pi}{15}(t-13) = -\frac{\pi}{2}$

$t - 13 = -7.5$

$t = 5.5$ ✓

(d) Sketch the graph of $M(t)$ for $0 \le t \le 30$, showing any stationary points, points of inflection and intercepts with the axes. **2**



QUESTION TWENTY NINE (5 marks)

Let $f(x) = xe^{-x^2}$.

(a) Find the coordinates of the stationary points of $y = f(x)$. You do NOT need to check the nature of the stationary points. **3**

$f(x) = uv$ $u = x$ $v = e^{-x^2}$

$u' = 1$ $v' = -2x e^{-x^2}$

$f'(x) = x(-2xe^{-x^2}) + e^{-x^2}$

$= (1-2x^2)e^{-x^2}$ ✓

$f'(x) = 0$

$(1-2x^2)e^{-x^2} = 0$

$1-2x^2 = 0$

$x^2 = \frac{1}{2}$

$x = \frac{\sqrt{2}}{2}$ or $x = -\frac{\sqrt{2}}{2}$ ✓

When $x = \frac{\sqrt{2}}{2}$, $f\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}e^{-\frac{1}{2}} = \frac{1}{\sqrt{2e}}$

When $x = -\frac{\sqrt{2}}{2}$, $f\left(-\frac{\sqrt{2}}{2}\right) = \left(-\frac{\sqrt{2}}{2}\right)e^{-\frac{1}{2}} = -\frac{1}{\sqrt{2e}}$

∴ The stationary points are

$\left(\frac{\sqrt{2}}{2}, \frac{1}{\sqrt{2e}}\right)$ and $\left(-\frac{\sqrt{2}}{2}, -\frac{1}{\sqrt{2e}}\right)$ ✓

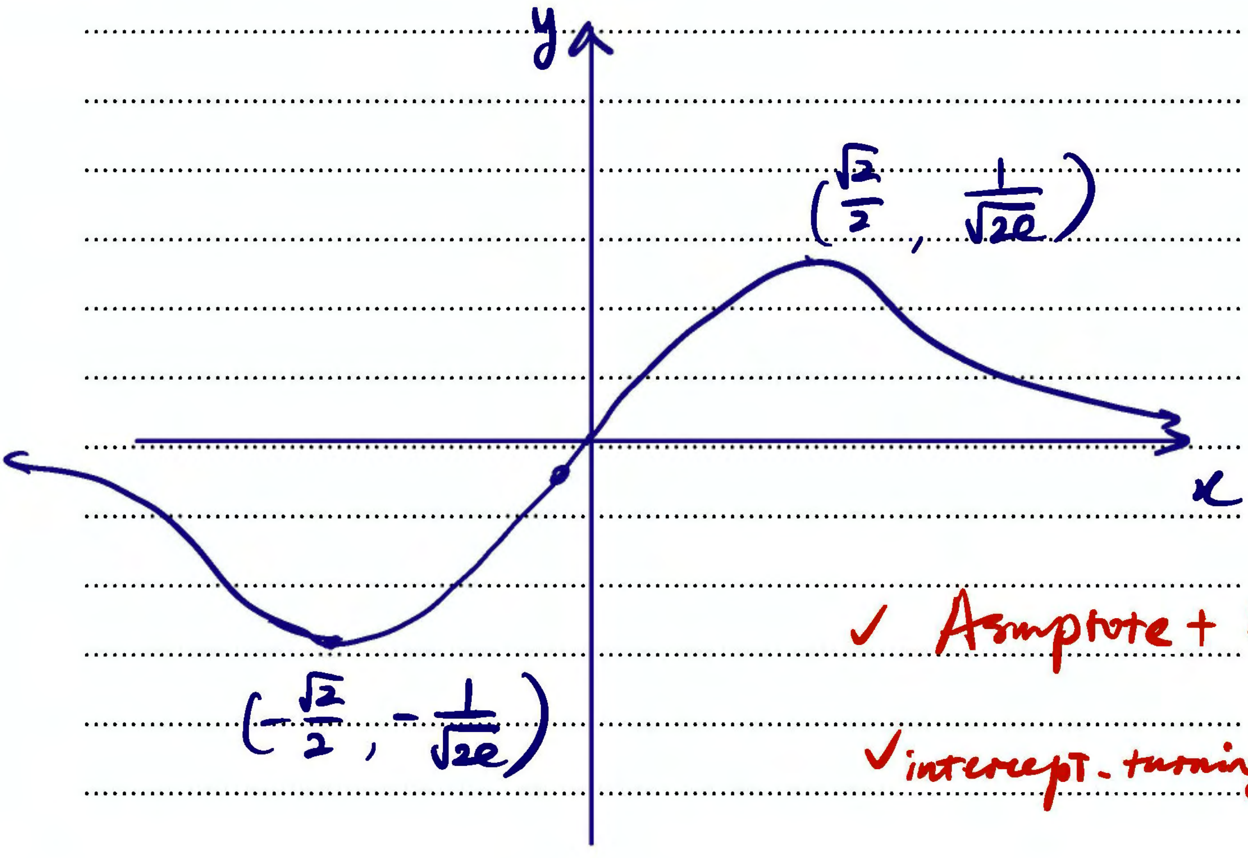
(b) Without using any further calculus, sketch the graph of $y = f(x)$, showing stationary points, any intercepts and asymptotes. **2**

Intercept $x=0$, $f(0)=0$

Asymptotes: $x \to \infty$, $f(x) = \frac{x}{e^{x^2}} \to 0^+$

$x \to -\infty$, $f(x) = \frac{x}{e^{x^2}} \to 0^-$

$f(x)$ is an odd function.



✓ Asymptote + Shape

✓ intercept - turning points

CANDIDATE NUMBER

Section II

Part D

QUESTION THIRTY (9 marks)

Jackie took a loan of \$500 000, which is to be repaid over 30 years, by 360 equal monthly repayments of $\$M$. The first repayment will be made one month after the loan is taken out. Interest on the loan is charged at 6% per annum, compounded monthly. Let A_n be the amount owing after the nth monthly repayment.

(a) Show that $A_2 = 500\,000(1.005)^2 - M(1 + 1.005)$. **2**

Must start w/ A_1

$r = \frac{6\%}{12} = 0.005$

$A_1 = 500000(1 + 0.005) - M = 500000(1.005) - M.$

$A_2 = A_1(1 + 0.005) - M$

$= (500000(1.005) - M)(1.005) - M$

$= 500000(1.005)^2 - M(1.005) - M$

$= 500000(1.005)^2 - M(1 + 1.005)$

(b) Show that $A_n = 500\,000(1.005)^n - M\left(\dfrac{(1.005)^n - 1}{0.005}\right)$. **1**

Must start w/ the geometric series.

$A_n = 500000(1.005)^n - M(1 + 1.005 + 1.005^2 + \cdots + 1.005^{n-1})$

$= 500000(1.005)^n - M\left(\dfrac{1.005^n - 1}{1.005 - 1}\right)$

$= 500000(1.005)^n - M\left(\dfrac{1.005^n - 1}{0.005}\right)$

(c) Show that the monthly repayment is \$2998, correct to the nearest dollar. **2**

$500000(1.005)^{360} - M\left(\frac{1.005^{360}-1}{0.005}\right) = 0$ ✓

$500000(1.005)^{360} - 200M(1.005^{360}-1) = 0$

$M = \frac{500000(1.005)^{360}}{200(1.005^{360}-1)}$ ✓

$= 2997.75\ldots$

$\approx \$2998.$

(d) Immediately after the 120th repayment, Jackie made a one-off payment, reducing the amount owing to \$350 000. The interest rate stays at 6% per annum, compounded monthly, and her monthly repayment remains \$2998.

(i) After how many more months will the amount owing be completely repaid? **2**

She needs to repay for n more months:

$350000(1.005)^n - 2998\left(\frac{1.005^n-1}{0.005}\right) = 0$ ✓

$350000(1.005)^n - 2998\times 200(1.005^n-1) = 0$

$(350000 - 2998\times 200)1.005^n = -2998\times 200$

$1.005^n = \frac{1499}{624}$

$n = \log_{1.005}\left(\frac{1499}{624}\right)$ ✓

$n = 175.72$

(ii) Find the amount of the last repayment. Give your answer correct to the nearest dollar. **2**

When $n = 175$.

$350000(1.005)^{175} - 2998 \times 200(1.005^{175} - 1)$

$= 2144.74\cdots$ ✓

After one month.

The last repayment $= 2144.74 \times 1.005$

$= 2155.46\cdots$

$\doteqdot \$2156$ ✓

Method 2.

When $n = 176$

$350000(1.005)^{176} - 2998 \times 200(1.005^{176} - 1)$

$\doteqdot -842.54$ ✓

$2998 + (-842.54)$

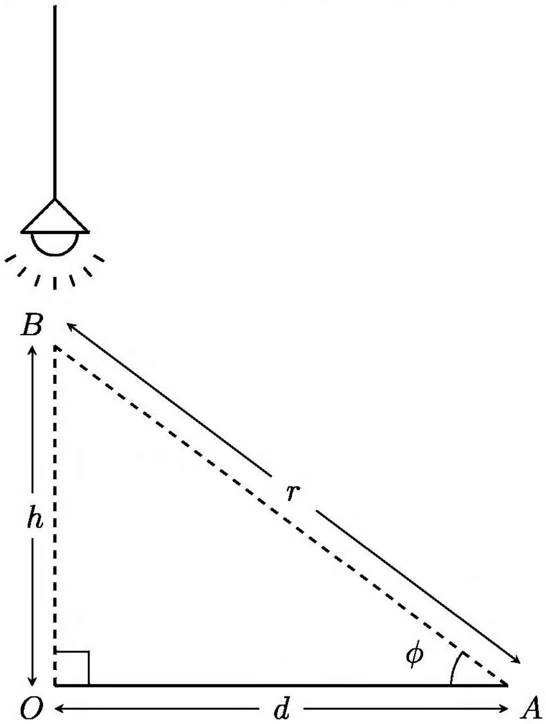
$\doteqdot \$2156$ ✓

QUESTION THIRTY ONE (5 marks)

The lamp shown in the diagram below can move along the vertical line OB using a pulley system. The point A lies on the horizontal line OA at a distance of d from O. The intensity I of light from the lamp at point A is inversely proportional to the square of its distance r from the lamp. The intensity I is also directly proportional to $\sin\phi$, where ϕ is the angle of elevation from A to the lamp. Hence, the formula for I can be expressed as: 5

$$I = \frac{C\sin\phi}{r^2}, \quad \text{where } C \text{ is a positive constant.}$$

Determine the height h of the lamp above the line OA that maximizes the intensity of light at the point A. Express your answer in terms of d.



$$I = \frac{C\sin\phi}{r^2}$$

$r^2 = h^2 + a^2$, $\sin\phi = \frac{h}{r} = \frac{h}{\sqrt{h^2+a^2}}$ ✓

$$I = c\left(\frac{h}{\sqrt{h^2+a^2}}\right)\left(\frac{1}{h^2+a^2}\right)$$

$$= ch(h^2+a^2)^{-\frac{3}{2}}$$ ✓

One mark for suitable substitution.

$$\frac{dI}{dh} = c(h^2+a^2)^{-\frac{3}{2}} + c\left(-\frac{3}{2}\right)(2h)(h^2+a^2)^{-\frac{5}{2}}h$$

$$= c(h^2+a^2)^{-\frac{5}{2}}\left[(h^2+a^2) - 3h^2\right]$$

$$= c(h^2+a^2)^{-\frac{5}{2}}(a^2-2h^2)$$

When $\frac{dI}{dh} = 0$. $a^2 - 2h^2 = 0$

$$h^2 = \frac{a^2}{2}$$

$$h = \frac{\sqrt{2}}{2}a \quad \left(\text{reject } h = -\frac{\sqrt{2}}{2}a\right).$$

When $h = \frac{1}{2}a$.

$$\frac{dI}{dh} = c\left(\left(\frac{a}{2}\right)^2 + a^2\right)^{-\frac{5}{2}}\left(a^2 - 2\left(\frac{a}{2}\right)^2\right)$$

$$= c\left(\frac{5}{4}a^2\right)^{-\frac{5}{2}}\left(\frac{1}{2}a^2\right) > 0$$

When $h = a$

$$\frac{dI}{dh} = c(a^2+a^2)^{-\frac{5}{2}}(a^2-2a^2) < 0$$

$\therefore$ When $h = \frac{\sqrt{2}}{2}a$, the intensity I is maximized.

QUESTION THIRTY TWO (7 marks)

Consider the functions $f(x) = 2^x$ and $g(x) = 3^x$.

(a) Find a single transformation that transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. **2**

$f(x) = 2^x$

$g(x) = \left(2^{\log_2 3}\right)^x$

$= 2^{(\log_2 3)x}$ ✓

Horizontal dilation by a factor of $\frac{1}{\log_2 3}$ ✓

(or $\log_3 2$)

(b) A new function $h(x)$ is found by taking the graph of $y = -g(-x)$ and translating it up by b units, where $b > 1$.
Sketch the graph of $y = h(x)$ showing the x-intercept and the asymptote in terms of b. **2**

$-g(-x) = -3^{-x}$

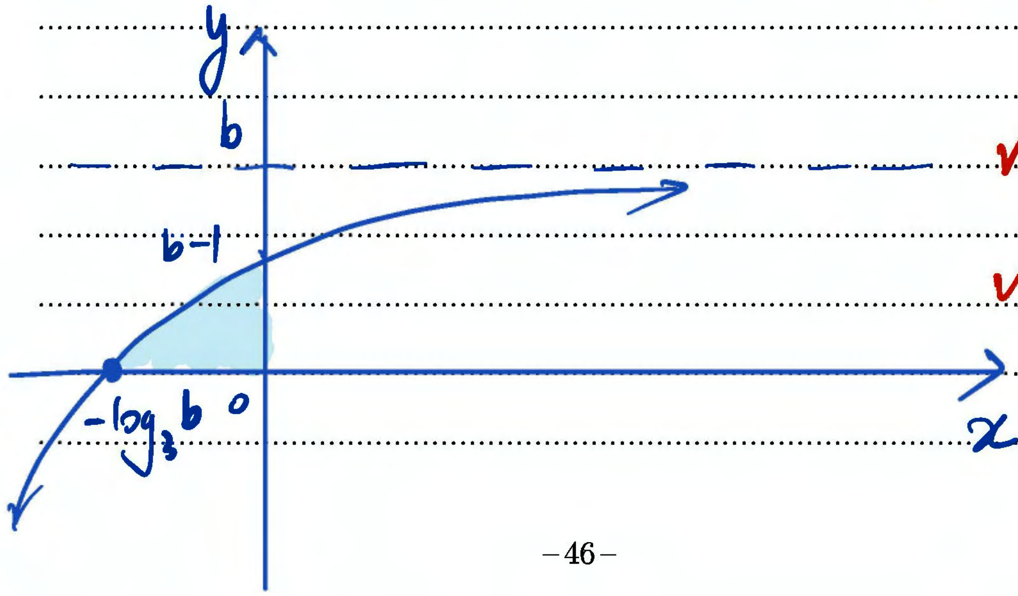
$h(x) = -3^{-x} + b = -\left(\frac{1}{3}\right)^x + b$

For y-int:

$x = 0, \quad h(0) = b - 1$

For x-int:

$-\left(\frac{1}{3}\right)^x + b = 0. \quad b = \left(\frac{1}{3}\right)^x. \quad x = \log_{\frac{1}{3}} b = -\log_3 b$



✓ Intercepts.

✓ Shape + asymptote.

(c) Hence find the value of b such that the area bounded by the curve $y = h(x)$, the x-axis and the y-axis is exactly $\dfrac{1}{\ln 3}$ square units. **3**

$\int_{-\log_3 b}^{0} (-3^{-x}+b)\, dx = \frac{1}{\ln 3}$

LHS $= \left[\frac{3^{-x}}{\ln 3} + bx\right]_{-\log_3 b}^{0}$ ✓ primitive.

$= \left(\frac{3^0}{\ln 3} + 0\right) - \left(\frac{3^{\log_3 b}}{\ln 3} - b\log_3 b\right)$

$= \frac{1}{\ln 3} - \frac{b}{\ln 3} + b\log_3 b$ ✓

LHS = RHS.

$\frac{1}{\ln 3} - \frac{b}{\ln 3} + b\log_3 b = \frac{1}{\ln 3}$

$-\frac{b}{\ln 3} + b\log_3 b = 0.$

$b\log_3 b = \frac{b}{\ln 3}$

$b \neq 0,$ $\log_3 b = \frac{1}{\ln 3}$

$\frac{\ln b}{\ln 3} = \frac{1}{\ln 3}$

$\ln b = 1$

$b = e.$ ✓

END OF PAPER